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ABSTRACT

This report describes a program for increasing adolescent visualization and understanding of geometry problems. The targeted population consisted of high school students in two geometry classes in a growing middle-class community. The inability of students to adequately visualize and understand geometry problems was documented through teacher-made tests and quizzes, student journals, and teacher journals. Analysis of probable cause data revealed poor performance on middle school geometry problems, lack of motivation to do the work, teacher observation of student inability to visualize the spatial situation, and parent comments substantiating the student stress level of not seeing the problem. After a review of the current geometry curriculum, it was discovered that an over-emphasis was placed on memorization and information giving, and there was a lack of hands-on instruction with the use of manipulatives to develop the visual skills. A review of solution strategies suggested by experts in the field of mathematics combined with an analysis of the problem setting resulted in these interventions. Materials that support visual focusing were created for hands-on use by students. An increase in student involvement and cooperative learning were used to foster understanding and thinking skills. Project results found that based on the presentation and analysis of the data on hands-on cooperative learning, students showed a more positive attitude towards math and a desire to work with partners or in cooperative groups. Students also indicated a preference for using hands-on learning and the use of manipulatives rather than using traditional learning methods. Test scores showed improved grades when students worked with a partner or in a group; however there was no marked difference in scores when students worked alone. Contains 19 references. (Author/NB)

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ED 422 179

DOES THE USE OF HANDS-ON LEARNING, WITH MANIPULATIVES, IMPROVE THE TEST SCORES OF SECONDARY EDUCATION GEOMETRY STUDENTS

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An Action Research Project Submitted to the Graduate Faculty of the School of Education in Partial Fullfillment of the Requirements for the Degree of Master of Arts in Teaching and Leadership

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DOES THE USE OF HANDS-ON LEARNING, WITH MANIPULATIVES, IMPROVE THE TEST SCORES OF SECONDARY EDUCATION GEOMETRY STUDENTS

ABSTRACT

This report describes a program for increasing adolescent visualization and understanding of geometry problems. The targeted population consists of high school students in two geometry classes in a growing, middle class community. The inability of today's students to adequately visualize and understand geometry problems will be documented through teacher-made tests and quizzes, student journals and teacher journals.

Analysis of probable cause data reveals poor performance on middle school geometry problems, lack of motivation to do the work, teacher observation of student inability to visualize the spatial situation and parent comments substantiating the student stress level of not seeing the problem. After a review of the current geometry curriculum, it was discovered that an over-emphasis was placed on memorization and information giving, and there was a lack of hands-on instruction with the use of manipulatives to develop the visual skills.

A review of solution strategies suggested by experts in the field of mathematics, combined with an analysis of the problem setting, resulted in these interventions. Materials that support visual focusing will be created for hands-on use by students. An increase of student involvement and cooperative learning will be used to foster understanding and thinking skills.

The results of my project found that based on the presentation and analysis of the data on hands-on, cooperative learning, the students showed a more positive attitude toward math and a desire to work with partners or in cooperative groups. The students also indicated a preference for using hands-on learning and the use of manipulatives rather than using traditional learning methods. Test scores showed improved grades when students worked with a partner or in a group. However, there was no marked difference in scores when students worked alone.

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CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

The students of the targeted two sophomore geometry classes exhibit frustration and difficulty in visualizing and understanding high school geometry problems. Evidence for the existence of the problem includes low grades on middle school geometry problems, teacher observation of students' inability to visualize the spatial situation, and parent comments substantiating the student stress level of not seeing the problem.

Immediate Problem Context

The subject site is a high school located in a western suburb of a large Midwestern city. It has a total enrollment of 2,037. There are a total of 19 schools in this unit district: 13 elementary schools, 4 middle school, and 2 high schools. The site's population comes from two of the district's middle schools. From the total number of students at this site, the following is a breakdown of ethnic characteristics: 87.8% White, 2.9% Black, 3.1% Hispanic, 6.1% Asian/Pacific Islander and 0.1% Native American. The subject site has an attendance rate of 96.5% with a chronic truancy rate of 0.6%. There is a rate of 8.6% student mobility. The site has an average class size of 26.5 students. The total number of teachers at this site is 109 and there are 5 administrators at this location (School Report Card, 1997). These figures are subject to change in the very near future due to a new housing development about to be completed in one of

the feeder communities. Projected figures are stating that there be at least 300 more students in this building by the year 2000 (School District ___ Enrollment Projections). This will create tremendous overcrowding in the site school. A referendum is currently being drafted to address this upcoming problem.

The average teaching experience for the unit district containing the site is 13.9 years with 58.3% of the total number of teachers with a Master's Degree or above. The unit district has a total number of 749 teachers; 74.6% are female, while 25.4% are male. The district's teacher racial/ethnic background breaks down as follows: 97.3% White, 1.3% Black, 0.7% Hispanic, 0.7% Asian/Pacific Islander and no Native Americans. The average teachers' salary in the site's district is $48,867, while the average administrators' salary is $71,769 (School Report Card, 1997).

The subject site's graduation requirements include 21 Carnegie Units (CU) with a semester course equaling one-half CU. The mathematics graduation requirement is two CUs, which usually includes one CU of geometry (School Report Card, 1997).

The site school was built in 1972-1973 when open classrooms were the educational trend. Most of the classrooms in the school, including the classroom used for this study, have no windows. It is a relatively large, yet overcrowded, school. The classroom itself is bright, roomy and has dry erase boards rather than chalk boards. However, because of poor ventilation, the classroom temperature tends to get very hot.

The geometry curriculum used for these classes is the same curriculum mandated for all geometry classes within the district. The text used is Geometry by Scott, Foresman published in 1990. In addition to the text, the subject matter presented in the classroom is supplemented by teacher-constructed materials.

The Surrounding Community

As stated above, the subject site is located in a western suburb. The student population of this site is drawn from two communities, both of which are located in the same county. The most

recent census showed a population of 51,464 residents in Community 1. According to Living in Greater _____, 1995, the median age of Community 1 is 33 years and the average income is $72,171. The average home value in this community is $201,113 and the average apartment rent is $734. This community prides itself as being a family-oriented community. The high school students from half of this community attend the site school, while the other half of the students attend the second high school in the unit district.

The most recent census showed a population of 11,390 residents in Community 2. Living in Greater _____, 1995, states that the median age of this community is 30.5 years and the average income is $67,503. The average home value of Community 2 is $141,639 and the average apartment rent is $624. This is an up-and-coming community with a new 650 acre development about to be completed. Approximately 344 rental housing units are part of this project. Any high school students residing in this development, and in this community, also attend the site school. As stated above, upon completion of this development, projections indicate overcrowding of the site school. A referendum is currently being written to address this issue. If it fails, alternative measures (i.e. split shifts,etc.) will be investigated to deal with the overcrowding situation.

National Context of the Problem

Geometry learning begins when children start to see and to know the physical world around them, and it can continue to very high-level geometrical thinking through inductive processes or within deductive systems (Kilpatrick & Nesher, 1990). According to Daniels, Hyde and Zemelman (1993), the concepts of geometry and measurement are best learned through experiences that involve experimentation and the discovery of relationships with concrete materials. There is a definite need in education today to make the study of geometry meaningful and real-world. The text book should be just one of the means to accomplish this goal. Another method to be used would be the use of manipulatives including three-dimensional objects. The 1995 National Council of Teachers of Mathematics Standards state the need for a shift in the vision of learning mathematics toward investigating, formulating, representing, reasoning, and

applying a variety of strategies to the solution of problems-then reflecting on these uses of mathematics-and away from being shown or told, memorizing and repeating. The NCTM Standards further state that as schools and teachers change their practices, they face the dilemma that the result of their efforts to meet new goals may not be supported by traditional assessment practices because such practices are inconsistent with these new views of mathematics and how learning progresses.

CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document the problem that students exhibit frustration and difficulty in visualizing and understanding high school geometry problems, the targeted classes were given a survey, developed by the researcher (Appendix A), to record their attitude and feelings toward math. The parents of these students were contacted via letter (Appendix B) regarding their student's participation in this project, and were also given a survey (Appendix C) to give their opinions regarding their child's math abilities. Middle school math teachers were interviewed regarding students' past performance levels in geometry. The above documentation was gathered during the first two weeks of the semester. The student survey was given to the two targeted sophomore geometry classes. A total of 47 students took the survey, with group one consisting of 23 students and group two having 24 students.

It is interesting to note, in Table 1, that 55% of students surveyed showed a dislike for math. This, in addition to other incongruities reflected in the survey, are the basis for this researcher's hypothesis. Eighty five percent of the students felt math is hard to understand and 76% felt this particular class was going to be hard. Yet, 94% of the students expected to achieve an A or a B in the class. When asked if they generally got good grades in class, 68% responded yes. However, in the previous semester of math, only 58% achieved an A or a B. These facts either indicate a positive attitude for these students, or the fact that they have grown to consider a C as a good grade in math. Ninety percent of the students achieved a C or better in their previous semester.

Table 1

Results of Student Survey

Question	Yes Responses	No Responses
1. I like math.	45%	55%
2. I think math is often hard to understand.	85%	15%
3. I think it's important for me to study math.	79%	21%
4. I think it's important for all students to study math.	66%	34%
5. I feel that math is useful to me right now.	40%	60%
6. I feel that math will be useful to me in the future.	83%	17%
7. I am looking forward to this geometry class.	59%	41%
8. I think this class is going to be hard.	76%	24%
9. I have trouble visualizing three dimensional objects.	15%	85%
10. I like to do hands-on work.	81%	19%
11. I like to work with a partner.	77%	23%
12. I like to work in cooperative groups.	70%	30%
13. My math homework is usually interesting.	21%	79%
14. I generally get good grades in my math class.	68%	32%
15. I have been introduced to geometry in the past.	72%	28%
16. What grade did you receive in second semester algebra?	A - 9% B - 49% C - 32% D - 9% F - 2%	
17. What grade do you expect to achieve in this class?	A - 37% B - 57% C - 7%	

Another question which raises concern regarding the motivation of students in math is that only 21% of the students consider their math homework interesting. Traditionally, math students

have lamented, "Why do I need to study math? I'll never use it." In fact, only 40% of the students surveyed felt math is useful to them right now. Even though 83% recognize math will be useful to them in the future, it is important for students to learn their math now. Like building blocks, a good foundation is necessary to tackle the more difficult math principles which lie ahead. Eighty-five percent of the students responded that they could visualize three dimensional objects.

Parents were given a survey on Teacher-Parent Curriculum Night during the second week of school. A total of 28 parents, from both classes, participated in the survey.

Table 2

Results of Parent Survey

Questions	Yes Responses	No Responses
1. My student likes math.	64%	36%
2. My student is motivated in math class.	64%	36%
3. My student sees a use for math in the future.	68%	32%
4. My student can visualize and draw a three-dimensional object.	88%	12%
5. My student likes to do his/her homework.	43%	57%
6. In the past, my student has been taught math using cooperative group learning.	43%	57%
7. My student is a male.	54%	46%
8. My student is looking forward to geometry this year.	67%	33%
9. My student studies for math quizzes and tests.	86%	14%
10. Please circle the grade your child received in second semester algebra last year.	A - 11% B - 39% C - 25% D - 18% F - 7%	

Probably the most interesting fact revealed by these surveys is that 55% of the students showed a dislike for math. A much smaller percent (36%) of the parents, however, recognized this apprehension in their own children.

Results of Teacher Interviews

During the first two weeks of school, five middle school math teachers were interviewed (Appendix D) regarding students' past performance levels in geometry. Each of the teachers interviewed was employed with in the same district as the site high school. One teacher is currently a high school math teacher, but had been a middle school math teacher and math chairperson for 22 previous years. This teacher has been very involved in interaction with math teachers from this district and other neighboring districts within the state.

In interviews with these middle school math teachers, it was determined that past performance levels of students reflected a lack of interest and ability to visualize and understand geometry problems. Traditional teaching methods failed to motivate students and quite frequently bored them to distraction. Many middle school teachers were ill-prepared or unwilling to use a constructivist approach to learning mathematics. Most of the teachers agreed that some form of combination between traditional teaching methods and a constructivist approach produced the best results in helping middle school students learn math principles. However, there was no consensus as to what this exact combination should be. Similarly, these middle school math teachers agreed that the use of calculators was important but the degree of their use was debatable. The teachers felt that many students showed an inability to read and understand a math book. They felt also that students'opinions of math in general were often tainted by their parents' own apprehension and phobias concerning math.

Probable Causes

Site-Based

This researcher is concerned with the low motivation of students in math and feels that a more constructivist approach to teaching would improve motivation and increase test scores. These

same concerns are routinely addressed in math department meetings at the site school. It is often discussed as to what methods or alternative course materials can be used to increase learning and improve motivation. Math teachers at the site school mirrored the opinions of the middle school teachers regarding the merits of using traditional teaching versus hands-on learning with manipulatives. The majority of the math teachers at the site school favored traditional teaching methods. Similarly, teachers at the site school conflicted regarding the use of technology in the classroom. Some teachers strongly support a very calculator driven curriculum, while others do not believe in the use of calculators in the classroom. The use of calculators by site based math teachers has only recently increased. Approximately one-half of these teachers allow the use of calculators in the classroom.

After conducting student and parent surveys of the targeted classes, there was evidence to support the concerns of this researcher. Thirty-six percent of the parents, in reference to their child, and 55% of the students themselves, expressed a dislike for math. Previous teaching methods, as well as the textbooks used to present the material, contributed to creating an atmosphere of low motivation, boredom and difficulty in understanding math concepts. The generally wide-spread pre-conceived perception that females are inferior to males in performing and understanding math problems may also contribute to the problematical attitudes previously mentioned.

Literature-Based

"Geometry learning begins when children start to see and to know the physical world around them, and it can continue to very high-level geometrical thinking through inductive processes or within deductive systems" (Kilpatrick & Nesher, 1990, p.93). Such visualization is important because it enhances the mental processes involved in mathematics. Math classes of the past were commonly based on rote memorization of facts, teachers lecturing and students working countless problems from the book. It is probably no surprise that many students lose interest in math due to this artificial environment. "The successive elimination of children at each grade level from

enjoying and understanding mathematics is like a strainer that allows only a few to continue with confidence and power. By high school, when students can start avoiding math courses, each year only about half continue" (Daniels, Hyde & Zemelman, 1993, p.69).

Many math textbooks in the past were not conducive to constructivist teaching styles. "Most teachers rely heavily on the textbook for their mathematics lessons. Since there is very little alternative material available, teachers are left with the enormous burden of redesigning their own mathematics curricula" (Fosnot-Twomey & Schifter, 1993, p.13). Other causes also contribute to this dilemma. Older, overburdened or indolent teachers are often set in their ways and are unwilling to experiment with, or implement, new teaching styles. The amount of time involved in this approach to teaching discourages many teachers from deviating from textbooks which contain traditional homework problems and tests prepared by the editor.

Pre-conceived notions concerning math, especially by female students, have lead to a socially acceptable position that it's permissible for girls to be weak in math. "Reality shows that females' social learning and beliefs about themselves with regard to mathematics have been particularly detrimental both to society as a whole and to females as a group" (Fennema & Leder, 1990, p.1).

CHAPTER 3

THE SOLUTION STRATEGY

Literature Review

There are many solutions to the problem of student frustration in visualizing and understanding high school geometry problems. The use of manipulatives, cooperative learning and hands-on learning styles will be investigated as solutions to the above stated problem. It is also hoped, as a by-product of implementing these methods, to improve the student motivation level in math and actually have them enjoy the learning process experience.

Motivate Learning

It is necessary to create a geometry climate that will stimulate and motivate students to learn. In order to stimulate students more in math, we need to attack the classroom environment and make it more motivationally oriented. According to Fosnot-Twomey & Schifter (1993), if the environment that we set up for learning is meaningful to them (the students), they will be motivated enough so that they will try to create meaning out of whatever it is that we are presenting to them. The age old problem of how to motivate students still exists. Below, several methods are discussed which may help in stimulating learning by keeping the students attention better and by actively involving them in the learning process.

Manipulatives

The use of manipulatives, particularly in geometry, can create a level of excitement and enjoyment for the students. Manipulatives are a form of hands-on activity. They would include using physical objects to illustrate geometrical formations and relationships. Examples of such

physical objects would be geoboards, cardboard pieces to illustrate intersecting planes and tanagrams. According to Daniels et al. (1993):

> The concepts of geometry and measurement are best learned through experiences that involve experimentation and the discovery of relationships with concrete material. When students construct their own knowledge of geometry and measurement, they are more able to use their initial understandings in applied, real-world settings. They develop their spatial sense in two or three dimensions through explorations with real objects (p. 76).

The purpose of manipulatives would be to allow students to learn a geometric principle in more than one way. In other words, instead of just hearing about a math principle, they also get to see it and feel it. When used, Ernest (1994) reports that students are more willing to participate and experiment in math projects, thus raising their self-confidence in their math ability. Sowell's study, as cited by Ernest, "concluded that mathematics achievement is increased by the long term use of manipulatives and that student attitudes toward mathematics are improved when they are instructed with manipulatives" (Ernest, 1994, p. 2).

Cooperative Learning

Crucial to a constructivist strategy is the need for change in teaching methods in the geometry classroom. The 1995 NCTM Standards suggest a decrease in whole-class, teacher-directed instruction and an increase in experiential, cooperative, hands-on learning. This is a far more active learning style and will be accompanied by noise and movement of students as they talk and collaborate. As Fosnot-Twomey and Schifter (1993) noted, changing your whole understanding and the basis on which you have been teaching all these years - it's scary! "Change is constant. If teachers are doing something in the same manner they did ten years ago, with all the advances in technology and research, there probably exists a new or improved method of performing this task" (DeVito, Krochover & Steele, 1993, p. 3).

By utilizing cooperative learning, students learn to work at problem solving as a group. This approach is similar to working styles used in corporate settings where groups or departments are

given a problem and expected to work together to an agreeable resolution. Members of the group feed off each other, and are often challenged by their peers in ways that a teacher could not do. Life experiences of various group members also may allow a teacher to look at a particular problem in a different, or novel, way.

Hands-On Learning

It is generally agreed that students learn in different ways. "As often as possible, school should stress learning that is experiential. With mathematics it means working with objects - sorting, counting and building patterns of number and shape; and carrying out real-world projects that involve collecting data, estimating, calculating, drawing conclusions and making decisions" (Daniels et al., 1993, p.9). Gardner (1993) has expounded on the existence of multiple intelligences and the need to address these intelligences when teaching children. Armstrong (1994) supports Gardner's theory when he says that students who show signs of bodily-kinesthetic intelligence should have opportunities to learn by manipulating objects or by making things with their hands. Hands-on learning with manipulatives should also appeal to the visual-spatial learner as well as the mathematical-logical student. Mathematical intuition should be nurtured, and students should naturally understand that mathematics emerges from their everyday experience (New Jersey State Department of Education, 1990). Group problem-solving projects should be encouraged as students look more to their peers and the technologies available to them in finding solutions.

Gender Issues

"Researchers have repeatedly observed that, beginning at or before puberty, males tend to score higher than females on measures of spatial skills" (Fennema & Leder, 1990, p.44). At both the middle school and high school levels, females reported lower levels of confidence in their ability to learn mathematics than did males (Fennema & Leder, 1990). It is hoped the above described solutions should improve the performance of not only the males but also the females in

the classroom. An additional suggestion to help the female student was offered by Berliner and Casanova (1996):

> Girls also achieved better in mathematics when they received more praise and prompting from the teacher. You can help bolster female students' math confidence by providing specific praise for good work, and "prompts" that encourage them to pursue difficult problems rather than give up or seek teacher help (p.200)

Summary

In order to give meaning to math teaching, students are best served by learning concepts by actual manipulation of physical materials. Motivation is best accomplished when there is an active involvement with physical objects. "The sole use of verbal techniques with children leads to what Piaget refers to as pseudo-learning, learning without meaningful understanding. The manipulation of physical objects often leads to scientific types of problem solving behaviors" (Carin & Sund, 1975, p. 338).

The most important thing for mathematics students is to understand mathematical ideas. This can be done by promoting dialogue and reflection on the students' part. A good teacher will help students become mathematically powerful through the use of creative hands-on activities that promote explanation, and well-functioning cooperative groups. Constructivist teachers encourage students to engage in dialogue, both with the teacher and with one another. One very powerful way students come to change or reinforce concepts is through social discourse. Student-to-student dialogue is the foundation upon which cooperative learning is structured (Brooks & Brooks, 1993). It is equally important for students to reflect on their findings and the various approaches used to arrive at those findings. "Effective learning is balanced with opportunities for reflection. Kids need time set aside for reflection. They need to become consciously aware of its power and their ability to use it. Adding reflective thinking to learning is one of the simplest of all instructional innovations" (Daniels et al., 1993, p.9). Learning logs or student journals are a good outlet for student reflection. This is substantiated by Daniels et al

(1993) when he says "as a cognitive tool, learning logs can help learners in any content field" (p.148). These ideas were also reiterated by Carpenter, Fennema and Lamon (1991):

> The good teacher is one who has sophisticated knowledge of the mathematics content being taught; who believes that mathematics is more than the rote memorization of procedures and facts but is also an evolving and generative form of conceptually based knowledge; who helps children connect what they know to that content; and who employs a variety of instructional strategies in achieving instructional goals (p. 32).

"Math does not have to bore," says Disney Math Teacher of the Year, Pat Taylor (Chapman, 1993, p.88). Students from Taylor's classes report on a regular basis that her approach, which begins with hands-on problem solving, uses cooperative groups, and focuses on the thinking skill applications, are "fun, interesting, difficult, and a place where math is real" (Chapman, 1993, p.88).

Project Outcomes

As a result of geometry instruction using manipulatives, cooperative learning and hands-on methods, the targeted groups will increase their visual-spatial skills and will report more interest and enjoyment when learning is done through these methods rather than traditional teaching styles. The students will be visibly more active in class and develop more self-confidence in their math skills.

The target group consists of two regular track sophomore geometry classes. One class has 24 students and the enrollment of the other class is 25 students. The learning project will be conducted during the period of September, 1997 through December, 1997 and will cover the first four chapters of their geometry book. The process of measurement to be used will be student pre-surveys and post-surveys, parent surveys, teacher-constructed quizzes, teacher-constructed tests, review of student journals and teacher observations recorded in a journal.

Solution Components

In order to accomplish the terminal objective, the following processes are necessary:

1. Write a letter to the parents of both groups of geometry students to inform them of the action research project taking place in their child's classroom.

2. Student pre- and post-surveys need to be created and administered to students for evaluative purposes.

3. Parent surveys need to be created to see how parents view their child's math skills and abilities.

4. Conduct interviews with middle school math teachers regarding students' past performance levels in geometry.

5. Materials that foster problem solving in visual-spatial areas of geometry will be developed.

6. A series of learning activities that address spatial geometry, hands-on learning with manipulatives, and cooperative learning will be developed for my geometry classes.

7. Curricular units reflecting these decisions will be constructed.

8. Teacher-made tests and quizzes will be constructed to be used for evaluative purposes.

9. Students will weekly record in journals their impressions of these learning processes.

10. The teacher will weekly record in a journal observations regarding student participation, enthusiasm and ability to grasp the subject material.

Project Action Plan

Chapter one, which introduces all the core geometry principles such as point, line, plane, ray, segment, angle, etc., will be presented using two different teaching styles. One class will be the control group and will be taught by traditional teacher lecture methods for the entire chapter. During this time, the other class will be taught the same chapter using manipulatives, hands-on activities and cooperative learning. This second class will be called the experimental group. Both groups will be given the same evaluative tools during the chapter. However, the experimental group will take their chapter one quizzes with a random partner and the control group will take their quizzes for the chapter alone. Both groups will take the chapter one test individually. The following three chapters will be taught using manipulatives, hands-on activities and cooperative

learning to both classes in the same fashion. Thus, the progress of the experimental group versus the control group will be able to be measured for chapter one, where different teaching methods will be used. For the following chapters where the same method will be used for both groups, student progress will be measured in a similar fashion. Differences between the two groups, during and following chapter one, will be able to be evaluated as to whether teaching methods created a difference in success levels. Additionally, through the use of journals, enjoyment and motivational levels will be monitored and evaluated.

I. Weeks 1-2: Establishing the Climate

A. Reflective and Metacognitive Activities

1. Send letter home to parents explaining action research project (Appendix B)
2. Conduct student pre-surveys about math (Appendix A)
3. Interview middle school math teachers regarding students' past performance levels in geometry (Appendix D)
4. Journal set up
5. Dividing into, and explaining, control and experimental groups

B. Curriculum Activities

1. Develop lesson plans for first chapter, on the introduction to geometry, for each group
2. Develop hands-on activities to compliment first chapter for experimental group
3. Develop evaluative materials for chapter one
4. Compare/contrast lesson plans and strategies between control and experimental groups
5. Administer first quiz for chapter one (Appendix E)

II. Weeks 3-4: Implementing Strategies

A. Reflective and Metacognitive Activities

1. Conduct parent surveys about their student's math abilities at Curriculum Night (Appendix C)
2. Journaling on survey results

3. Discussion of learning styles
4. Teacher observation and recording of results

B. Curriculum Activities

1. Administer second quiz for chapter one (Appendix F)
2. Administer test for chapter one (Appendix G)
3. Develop lesson plans for chapter two, on reasoning in geometry, for each group

III. Weeks 5-6: Adapting to Different Styles

A. Reflective and Metacognitive Activities

1. Journal on chapter one strategies
2. Inform both groups that they will be receiving the hands-on with manipulatives, cooperative learning style of teaching for the remainder of the experimental time.
3. Select new partners and cooperative groups
4. Teacher observation of activities and recording of events
5. Analyzing and discussion of data

B. Curriculum Activities

1. Develop hands-on, cooperative group lessons on logic (chapter two) and secure necessary manipulatives to be used with these.
2. Develop evaluative tools for chapter two for each group
3. Administer partner quiz on chapter two for each group (Appendix H)
4. Teacher observation and reflection on student involvement and enjoyment of these learning styles.

IV. Weeks 7-10: Continuation of Constructivist Style

A. Reflective and Metacognitive Activities

1. Journal on chapter two strategies
2. Teacher observation of activities and recording of events
3. Analyzing and discussion of data

4. Teacher observation and reflection on student motivation in doing chapter three proofs in this fashion

B. Curriculum Activities

1. Administer second partner quiz for chapter two (Appendix I)
2. Administer test for chapter two (Appendix J)
3. Develop hands-on, cooperative group lesson plans for chapter three, on proofs
4. Develop evaluative tools for chapter three
5. Administer partner quizzes for each group for chapter three
6. Conduct a proof-off in each class

V. Weeks 11-15: More Constructivist Teaching

A. Reflective and Metacognitive Activities

1. Journal on chapter three strategies
2. Teacher observation of activities and recording of events
3. Analysis and discussion of data for chapter three
4. Journal on chapter four strategies
5. Teacher observation of activities and recording of events
6. Analysis and discussion of data for chapter four

B. Curriculum Activities

1. Administer test for chapter three
2. Develop hands-on, cooperative group lessons with manipulatives for chapter four covering parallel lines, planes, triangles and polygons
3. Develop evaluative tools for chapter four
4. Administer partner quizzes for chapter four
5. Teacher observation and reflection on student involvement and enjoyment of these learning styles

V. Week 16: Culminating Activities

A. Reflective and Metacognitive Activities

1. Journaling on learning styles/experiences
2. Discussion of the hands-on with manipulatives, cooperative learning style of teaching/learning
3. Teacher observation and recording of all this information
4. Assessment of entire project

B. Curriculum Activities

1. Administer test for chapter four
2. Administer post-survey to both groups of students
3. Teach origami to all students
4. Have students create a three-dimensional origami ornament for our geome-tree in our classroom
5. Have an open house in our classroom for teachers, students and parents to come see the fruits of our hands-on efforts

Methods of Assessment

The project will be assessed by evaluating student performance on teacher-made tests and quizzes. Journals created by both the students and the teacher will be evaluated to determine various results of the project. Parent surveys of their child's math interest and skills will be used as an evaluative tool. Additionally, pre-surveys and post-surveys will be given to the students as a means to further assess the success of the program.

CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The terminal objective of the intervention addressed two targeted sophomore geometry classes. These classes exhibited frustration and difficulty in visualizing and understanding high school geometry problems. Evidence for the existence of the problem included low grades on middle school geometry problems, teacher observation of students' inability to visualize the spatial situation, and parent comments substantiating the student stress level of not "seeing" the problem.

The objective of this project was to increase the visual-spatial skills of the targeted geometry classes and have them report more interest and enjoyment in the learning of geometry. The students would be visibly more active in class and develop more self-confidence in their math skills.

Geometry instruction using manipulatives, cooperative learning and hands-on methods was used in delivering subject matter content. The target group consisted of two regular track sophomore geometry classes. Chapter one, which introduces all the core geometry principles such as point, line, plane, ray, segment, angle, etc., was presented using two different teaching styles. One class was the control group and was taught by traditional teacher lecture methods for the entire chapter. During this time, the other class was taught the same chapter using manipulatives, hands-on activities and cooperative learning. This second class was called the experimental group.

Geoboards were used by the experimental group to learn about points, lines, segments, intersecting lines and segments, and planes. The experimental group also used rectangular cardboard box dividers to help with understanding the concepts of intersecting planes and points of intersection. Students from this experimental group became human points, and with the aid of string, this group learned about collinearity and non-collinearity. They next added large pieces of construction paper to their human points and then studied what kinds of points determine a plane. The instructor presented acute, obtuse, right, complementary and supplementary angles, as well as linear pairs, in the format of a skit to this group. The skit ended with a song, sung by the instructor and students, which reviewed all the information presented in the skit.

Both groups were given the same evaluative tools during the chapter. However, the experimental group took their Chapter one quizzes with a random partner and the control group took their quizzes for the chapter alone. The first quiz for Chapter one can be found in Appendix E, the second quiz for this chapter in Appendix F. Both classes took the Chapter one test individually. This test is located in Appendix G.

The following three chapters were taught using manipulatives, hands-on activities and cooperative learning to both classes in the same fashion. A variety of instructional techniques was used in the process of teaching these chapters. For the most part, the techniques used were non-traditional methods.

Chapter two dealt with inductive and deductive reasoning and logic. In each class, the students were paired with one partner and investigated an assortment of numerical, alphabetical, picture, and logic puzzles meant to explore reasoning. They also were introduced to truth tables in this chapter.

In preparation for Chapter three on proofs and putting thoughts in logical order, the students learned to write programs for complementary and supplementary angles on their graphing calculators. Another technique used to promote logical thinking made use of good directions. The students were put into pairs and asked to write good, clear directions to a designated location in

the school. They were not allowed to use any room numbers, names of rooms or individuals occupying those rooms, or names of hallways in their directions. Each set of directions was coded by the instructor as to its location in the school. Several days later the pairs of students selected directions not written by them, followed them, and reported their destinations to the class. A class discussion on writing clear, logical directions ensued. Chapter three was on proofs in geometry. After the preparatory work discussed above, the students began this chapter by doing a variety of cut and paste activities in their cooperative groups. Complete proofs were given to them in scrambled order and they had to cut the statements and reasons apart and paste them in the correct logical order. Gradually, they advanced to writing the entire proof themselves. Another group activity in this chapter involved each group being given a blank overhead sheet and overhead pen and having to work through and present a proof to the class. A proof-off to earn group points was given as a culminating activity for this chapter.

Chapter four covered parallel lines, planes, triangles and polygons. In this chapter, students were given plastic straws, which they used throughout the chapter to model parallel lines cut by a transversal, triangles, and polygons. A paper activity for Chapter four involved each student drawing a triangle of their choice on paper and labeling the vertices A, B, and C. Next, they tore off these vertices, put them together and discovered they formed a line. Since they already had learned that a line is 180 degrees, they concluded that the sum of the angles in a triangle is 180. To discover the formula for the number of diagonals in a polygon, the students again returned to their groups. They were given toothpicks and gumdrops and were instructed to build polygons having three through eight sides. After adding diagonals to their polygons, they discovered the correct formula for the number of diagonals in any polygon.

In both groups, all quizzes for Chapters two through four were taken with a random partner. The chapter tests for these chapters were taken individually by all students. During the course of the intervention, students wrote journals responding to various stem statements, selected by the instructor, relating to the learning techniques used at that time. Periodically, class discussions

were conducted so students could also voice their feelings about the manner in which they were learning math. After the completion of Chapter two, the control group also had an additional discussion directed at comparing and contrasting the two instructional techniques presented in chapters one and two. The instructor recorded weekly journal observations regarding student participation, enthusiasm and ability to grasp the subject material.

The culminating assignment for this intervention was an out of class origami project. Each student was required to make two origami ornaments, out of holiday gift wrap, which were placed on the geome-tree we had assembled in the classroom. The students had a three week time frame in which to complete their work, outside of class, and receive points for the project. The instructor used the last day of school before Thanksgiving break to introduce the project. Packets, which detailed the different kinds of origami folds and helped clarify origami instructions, were given to each student. The instructor pointed out the location in the classroom of approximately 15 origami books that the classes could use to find patterns for their ornaments. Outside research for patterns, such as the library, bookstores, etc., was also acceptable. Students practiced folding origami paper and took time to begin browsing through the books to look for patterns, on this day. It was also announced that there would be a holiday open house in the classroom to show off the completed project. Invitations were made and folded by the students, using origami. Anyone who wanted to invite a parent, sibling, friend or teacher to the open house could take an invitation. The open house was held after school one day just before winter break. Punch and cookies were served and many students were on hand to explain the project to the teachers, administrators, parents and friends who attended.

Presentation and Analysis of Results

In order to assess the effects of the two different teaching styles used in Chapter one, the quiz and test scores for the experimental group and control group were compared. Recall that the experimental group, or class one, was taught using hands-on learning with manipulatives in a cooperative learning environment. This experimental group took both quizzes in this chapter with

a random partner. The control group, or class two, was taught Chapter one using traditional teacher lecture methods. The control group took the Chapter one quizzes individually. Both groups were comprised of 25 students. The results of the quiz one comparisons are in Table 3. The average grade for this quiz for the experimental group was 86.3%. The average grade on the same quiz for the control group was 79.2%.

Table 3

Results of Chapter One, Quiz One

Group	A	B	C	D	F
Experimental	37.5%	45.8%	8.3%	8.3%	0%
Control	20%	44%	24%	8%	4%

The intervention seems to have had a positive effect on the results of the experimental group's quiz grades. While the results were not dramatic, they still seem to produce more high grades and fewer low grades. This pattern continued into the next quiz, as seen in Table 4.

Table 4

Results of Chapter One, Quiz Two

Group	A	B	C	D	F
Experimental	91.7%	8.3%	0%	0%	0%
Control	60%	36%	4%	0%	0%

The intervention, once again, has seemed to have had a positive effect on the experimental group's quiz grades. In both classes, there were a large number of high grades. However, the number of A's in the experimental group far exceeded those in the control group. The average grade for the experimental group was 98% on this quiz. The control group's average grade on this quiz was 91.2%.

The test for Chapter one was given to both groups individually. Both groups also received the same review material for this test. Results of the Chapter one test for both groups can be found in Table 5. The experimental group's average grade for the Chapter one test was 79%. The control group averaged 78.24% on this same test.

Table 5

Results of Chapter One Test

Group	A	B	C	D	F
Experimental	25%	25%	25%	16.7%	8.3%
Control	28%	28%	24%	8%	12%

The remaining three chapters for this action research project were taught to both classes in the same fashion, using hands-on manipulatives and cooperative learning. During this time all students took quizzes with a random partner and chapter tests individually. Homework points and extra credit points were available to both classes on an equal basis. Table 6 shows the overall percentages for the two groups at the end of the intervention time, which was the end of the semester. The average semester grade for the experimental group was 82.1%. The average semester grade for the control group was 79.6%.

Table 6

Results of Semester One Grades

Group	A	B	C	D	F
Experimental	16%	60%	16%	8%	0%
Control	8%	56%	28%	4%	4%

The intervention appears to have had a positive impact on the difference in grades between the two groups when the experimental group was allowed to work with partners. When each

groups worked as individuals, the impact was lessened. By the end of the semester, in both groups, the number of grades is centered in the B and C range. The number of A's, although small, is still larger than the number of F's.

During the course of the semester, both the experimental group and the control group were required to maintain journals. The instructor would give a stem statement to the students. The students were then asked to respond with a paragraph or two expressing their feelings regarding the statement. The purpose of keeping these journals was to monitor the students' reactions to the various teaching methods and learning styles being used.

In general, the students' journal entries largely favored group learning and the use of hands-on activities. There did not seem to be any difference in responses between good and poor students. The students particularly liked working with a partner, especially in the instance when they were allowed to choose their own partner. Many students stated in their journal entries that, "Two heads are better than one. If I didn't know the information, my partner usually did or at least together we could figure it out." The few students whose journals reflected that they did not enjoy these types of learning styles, generally complained only about working with a partner. These complaints for the most part came from the better students who felt they were carrying their partner and getting nothing in return. There were no negative responses to the use of hands-on learning. Unanimously, the students felt this type of learning clarified complex math concepts and made learning math more fun. Comments regarding the origami project were also very favorable. A few students, however, expressed frustration at the difficulty of such a project or felt the project should have been conducted during class time.

The instructor also maintained a journal during the course of the semester. This journal consisted of reflections by the instructor regarding the students' performances and behavior as a result of the learning processes used. The most notable observation was that the motivation level and enthusiasm of the students greatly increased when hands-on activities were introduced. The

instructor also noted in journal entries that the use of interactive learning between students resulted in a higher level of comprehension and understanding.

The same student survey (Appendix A) was given at the beginning and end of the intervention.

Table 7

Results of Student Surveys (Yes Responses Only)

Question	Beginning Yes Responses	Ending Yes Responses
1. I like math.	45%	57%
2. I think math is often hard to understand.	85%	55%
3. I think it's important for me to study math.	79%	82%
4. I think it's important for all students to study math.	66%	70%
5. I feel that math is useful to me right now.	40%	41%
6. I feel that math will be useful to me in the future.	83%	84%
7. I am looking forward to this geometry class.	59%	48%
8. I think this class is going to be hard.	76%	48%
9. I have trouble visualizing three dimensional objects.	15%	25%
10. I like to do hands-on work.	81%	89%
11. I like to work with a partner.	77%	95%
12. I like to work in cooperative groups.	70%	91%
13. My math homework is usually interesting.	21%	20%
14. I generally get good grades in my math class.	68%	75%
15. I have been introduced to geometry in the past.	72%	84%

The above survey was given in order to document the changes in attitudes of the students with regard to their feelings about math. The results clearly show an improved attitude toward math. The results also show an increase in desire to do hands-on math with a partner or in a cooperative

group. The largest improvement shown in the survey was in the number of students who felt math was often hard to understand. However, students realized over the course of the intervention that visualizing three dimensional objects is not as easy as they originally thought.

Conclusions and Recommendations

Based on the presentation and analysis of the data on hands-on, cooperative learning, the students showed a more positive attitude toward math and a desire to work with partners or in cooperative groups. The students also indicated a preference for using hands-on learning and the use of manipulatives rather than using traditional learning methods. Test scores showed improved grades when students worked with a partner or in a group. However, there was no marked difference in scores when students worked alone.

Quiz one of Chapter one was given to the control group individually and to the experimental group with partners. The results showed more A's with the experimental group and fewer C's or F's with this group. This result is as one would expect when two students work on a quiz together rather than alone. Because the students were randomly paired, it happened that good students were often paired together and, in some cases, poor students were together. It is too premature to draw any type of conclusion from the results of this quiz alone.

Quiz two of Chapter one was given in the same fashion as quiz one. The results of this quiz, however, were even more dramatic with an inordinate number of A's in the experimental group. The control group also showed a large number of A's and B's. Clearly, the material covered in this quiz was easy to grasp for these students, thus resulting in the large number of high grades. The fact that students were working together in the experimental group resulted in the larger number of A's for that group. The teaching methods did not seem to have an impact on the learning of this material by the students. Again, no conclusions can be drawn at this time.

The Chapter one test was given individually to both groups. The results of this test are almost identical. This leads the teacher to believe that the teaching styles used in Chapter one did not materially effect the amount of information the students learned. The partner quizzes produced

higher grades due to the fact that students were able to work together. The hands-on, manipulative approach did not cause improvement in test scores.

From this point on, students in both classes were taught in the same fashion. All students took quizzes with a random partner and chapter tests individually for the remainder of the semester. The results of the semester one grades were similar between the two groups. The student ability level is rarely exactly proportional between two classes. The results of the semester one grades were consistent with the results of the first chapter test, thus confirming the conclusions reached above.

During the course of the semester, both the experimental group and the control group were required to maintain journals. It was in these journals that the most significant differences in teaching methods became evident. Students in the experimental group, during Chapter one, wrote of their satisfaction and enjoyment of the teaching styles being used. The control group, during this time, expressed more opinions of boredom with the traditional teaching methods presented to them. For the remainder of the semester, where both groups were taught using hands-on, cooperative learning methods, the student journals reflected more the opinions of the experimental group during Chapter one. In other words, they expressed satisfaction, for the most part, with the manner in which they were being taught. Many of the students stated that they enjoyed working with a friend, and often some students would enjoy the challenge of helping fellow students. Weaker students, rather than enjoying a free ride, would often work harder to avoid the embarrassment of not knowing the material. Overall, the students' feelings about geometry improved during the course of the semester, as reflected by their journal entries. This indicates a satisfaction with the learning styles being used.

During the course of the semester, the instructor also maintained a journal. The most notable observation was that the motivation level and enthusiasm of the students greatly increased when hands-on activities were used. The instructor found hands-on teaching methods more enjoyable and often was discouraged when forced to continue traditional teaching methods, as in Chapter

one. The instructor felt the control group was being deprived during Chapter one. Throughout the rest of the semester, the instructor noted the students' preference for a variety of teaching methods and use of hands-on activities. By the use of these methods, enthusiasm was able to be maintained at a higher level throughout the course of the intervention time. As the instructor would circulate around the room during a cooperative learning activity, it was gratifying to see and hear the amount of interactive learning being shared between students. Contrary to what one might expect, discipline was not a problem during these group learning sessions. While the classroom noise level was louder than a traditional classroom, the atmosphere was still conducive to learning. The culmination of the intervention period was an origami project, including the display of a geome-tree. Students were required to create two ornaments using origami, thus providing decorations for the tree. Student responses during class discussions about the origami were very favorable, as noted in the instructor's journal.

A survey was given to both groups at the beginning and end of the intervention. The purpose of the survey was to gauge student feelings toward math and styles of learning. The results clearly showed an improved attitude toward math. The results also showed an increase in desire to do hands-on math with a partner or in a cooperative group. The largest improvement shown in the survey was in the number of students who felt math was often hard to understand. Students realized over the course of the intervention that visualizing three dimensional objects is not as easy as they originally thought. These results are consistent with the observations noted in both the students' and the teacher's journal entries. It was gratifying to the instructor to see the improved attitudes of the students, but it was not surprising to note that certain attitudes, such as interesting homework, stayed the same.

In conclusion, the instructor would recommend the use of hands-on, cooperative learning in teaching geometry. While the results may not necessarily produce better test scores, the students enthusiasm for learning math seems to improve. This improved enthusiasm translates into more positive feelings towards math by the students and thus would hopefully encourage them to

continue with additional math courses. Caution should be noted that final grades may be slightly inflated due to the use of partner quizzes throughout the semester. The use of hands-on, manipulative learning requires a change from traditional lesson plans. Teachers should expect additional work in order to prepare for such teaching styles. The instructor would recommend the use of these teaching methods in geometry classes for the reason that it makes geometry more meaningful and enjoyable for the students. It is the instructor's intention to continue the use of hands-on, cooperative learning both the rest of this year and in the future. It is the instructor's opinion that the increased level of enjoyment by the students, the positive reinforcements created by this type of teaching, and the lack of boredom, both for teacher and student, caused by the variety of teaching methods, make the extra efforts worthwhile. The instructor would share the information gleaned from this research with others who may be interested in cooperative learning and would encourage colleagues to experiment with this type of teaching. The primary impact from the future use of such teaching methods is to hopefully develop in the students a desire to learn and a love of math.

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Appendices

Appendix A

GEOMETRY SURVEY

Please complete the following questionnaire and return it to . Circle "yes" or "no" to answer each question. Thank you for your cooperation.

1. I like math.	Yes	No
2. I think math is often hard to understand.	Yes	No
3. I think it's important for me to study math.	Yes	No
4. I think it's important for all students to study math.	Yes	No
5. I feel that math is useful to me right now.	Yes	No
6. I feel that math will be useful to me in the future.	Yes	No
7. I am looking forward to this geometry class.	Yes	No
8. I think this class is going to be hard.	Yes	No
9. I have trouble visualizing three dimensional objects.	Yes	No
10. I like to do hands-on work.	Yes	No
11. I like to work with a partner.	Yes	No
12. I like to work in cooperative groups.	Yes	No
13. My math homework is usually interesting.	Yes	No
14. I generally get good grades in my math class.	Yes	No
15. I have been introduced to geometry in the past.	Yes	No
16. What grade did you receive in second semester algebra?	_____	
17. What grade do you expect to achieve in this class?	_____	

Appendix B

Letter to Parents

September 2, 1997

Dear Parent:

This letter is to inform you that I am currently pursuing an Action Research Master's Program. What this means is that I will be involving your child as an active participant in a study meant to determine whether students learn geometry better using "hands-on" interactive teaching rather than traditional teaching methods. Your child will be exposed to both teaching methods, and be tested on each. The results of these tests will be published in the paper I am writing. I expect this alternate teaching method to last approximately six to nine weeks.

If you have any questions regarding this study, please feel free to contact me at -------.

Sincerely,

Appendix C

PARENT SURVEY

Please complete the following questionnaire and return it to . Circle "yes" or "no" to answer each question. Thank you for your cooperation.

1. My student likes math.	Yes	No
2. My student is motivated in math class.	Yes	No
3. My student sees a use for math in the future.	Yes	No
4. My student can visualize and draw a three-dimensional object.	Yes	No
5. My student likes to do his/her math homework.	Yes	No
6. In the past, my student has been taught math using cooperative group learning.	Yes	No
7. My student is a male.	Yes	No
8. My student is looking forward to Geometry this year.	Yes	No
9. My student studies for math quizzes and tests.	Yes	No

10. Please circle the grade your child received in second semester algebra last year.

A B C D F

Comments:

Appendix D

Interview with Middle School Math Teachers

During the first two weeks of school, five middle school math teachers were interviewed regarding students'past performance levels in geometry. Each of the teachers interviewed was employed within the same district as the site high school. One teacher is currently a high school math teacher, but had been a middle school math teacher and math chairman for 22 previous years. This teacher has been very involved in interaction with math teachers from this district and other neighboring districts within the state. Following are the questions presented to these teachers as well as a summary of their answers:

Question 1: Please comment on your students' ability to visualize and understand geometry problems.

Response: The teachers felt that the past performance levels of the students generally reflected a lack of interest in math and an inability to visualize and understand geometry problems. The teachers commented that the students often had an inflated opinion of their math abilities.

Question 2: Please comment on the use of traditional teaching methods in middle school math and the effect on the students.

Response: While traditional teaching methods have been successful in teaching students to memorize certain math concepts, they have failed in being able to motivate students and to inspire them to expand their math horizons. Often when traditional teaching methods are used, students become bored and inattentive. Disruptive behavior can be a byproduct of this inattentiveness. Most teachers agree that traditional teaching methods are necessary in learning certain math skills and principles.

Question 3: Please comment on the use of hands-on and manipulative learning techniques for middle school students.

Response: The response to this was varied. Many middle school teachers have not been trained in these approaches to learning mathematics. Hence, they were unwilling to take the

Appendix D (Continued)

additional time necessary to learn and to utilize this type of teaching method. Those who have implemented this newer technique have used it in conjunction with, rather than in place of, traditional teaching methods. These teachers reported when this combination was used, students seemed to grasp the math principles involved faster and with more understanding. An additional benefit in using activity based learning was more enthusiasm and fewer discipline problems.

Question 4: Please comment on your students' ability to self-teach, that is, can they pick up a math book and learn math principles simply by reading about them.

Response: Laughter!! With few exceptions, students are unable and unwilling to read and comprehend a math book.

Question 5: Please comment on feedback received from students' parents regarding math experiences.

Response: The teachers commented that parents often express their own apprehension and phobias concerning math at conferences. They ask that additional help be obtained from teachers or they ask for the name of a tutor because they can't help their child themselves.

Question 6: Please comment on the use of calculators in the classroom.

Response: There seems to be a sharp difference of opinion regarding the use of calculators in the classroom. Some teachers feel that technology is the wave of the future and students should be as proficient as possible in their use. Others feel strongly that rote memorization of math facts is important and that students will learn enough about technology in the future. Still others feel the use of calculators is important but students must be able to recognize ridiculous answers.

Question 7: Please comment on the assignment of homework in middle school math.

Response: For some students there is a correlation of completed homework and good grades. Certain students do all their homework and still don't perform well on math tests. While others do no homework and consistently get good grades on tests. When a creative homework

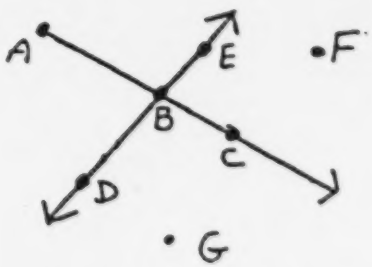
Appendix D (Continued)

assignment is given, there is a noticeable increase in the number of students who complete the assignment and more enthusiastic discussion in the classroom of the subject matter involved.

Appendix E

Geometry Quiz Chapter 1
Sections 1-1 through 1-5

1. Name the three undefined terms._____
2. Points or lines in the same plane are called._____
3. If two planes intersect, then their intersection forms a_____
4.



a. Name a line.
b. Name a ray._____
c. Name $\overrightarrow{DE}$ in another way._____
d. Name 3 line segments._____
e. Name 3 noncollinear points._____
f. Name 3 noncoplanar points._____
g. Name a pair of opposite rays._____

Answer the following True or False.

5. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ are the same line._____
6. $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are the same ray._____
7. $\overline{XY}$ and $\overline{YX}$ are the same segment._____
8. Three points always form a plane._____
9. Two points always form a line ._____

Draw a diagram using the following information.

10. Points A, B, and C are noncollinear.

11. Points A, B, X, and Y are collinear, where X is between A and B, and A is between X and Y.

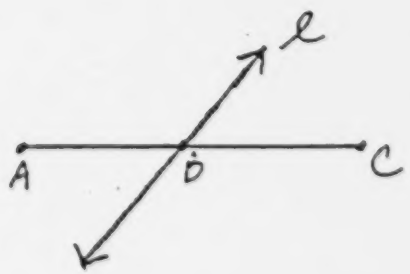
Appendix E (Continued)

12. Three coplanar lines that have two points of intersection.

13. X is on $\overrightarrow{YZ}$, but X is not on $\overline{YZ}$.

14. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays.

Fill in the blank based on the following diagram.



AB=BC

15. B is a· _____

16. ℓ is a _____

17. If AB=2x+9 and BC=5x-12, find x, AB and BC.

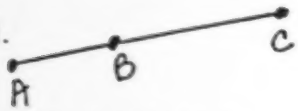
x=_____

AB=_____

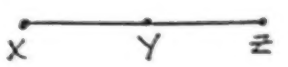
BC=_____

Appendix F

GEOMETRY CHAPTER 1 QUIZ 2 NAME_____

1. 

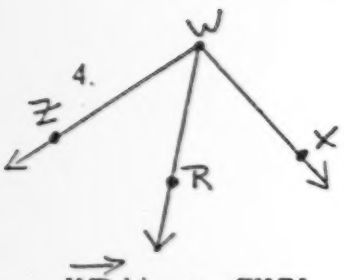
AB = 20, AC = 52
BC = _____

2. 

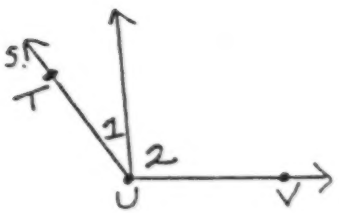
Y is the midpoint of $\overline{XZ}$
_____ = _____

3.

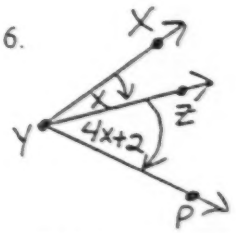
QR = 5x, RS = 3x + 1,
QS = 41
x = _____, QR = _____
RS = _____
Is R the midpoint of QS?___
(Yes or No)

4. 

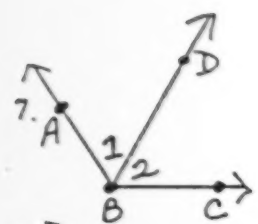
$\overrightarrow{WR}$ bisects <ZWX
m<_____ = m<_____

5. 

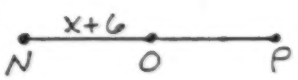
m<1 = 56, m<2 = 103
m<TUV = _____

6. 

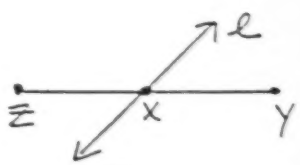
m<XYZ = x, m<XYP=72
m<ZYP = 4x + 2
x = _____, m<XYZ = ___
m<ZYP = _____

7. 

$\overrightarrow{BD}$ bisects <ABC,
m<1 = 56.
m<2=_____,
m<ABC = _____

8. 

NO = x+6, O is the
midpoint of NP, NP=32
x = _____, NO=_____
OP = _____

9. 

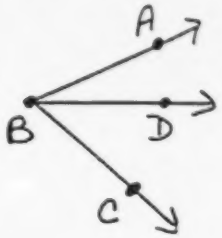
ℓ bisects $\overline{ZY}$ at X.
X is the _____
of ZY.

Appendix F (Continued)

10. **Draw a picture for this and solve.**

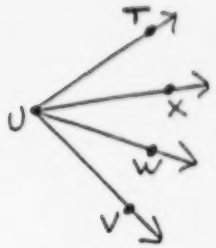
C is the midpoint of $\overline{AB}$. If AC = 18, AB = _____

11.



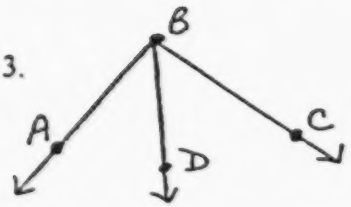
m<ABC = 62
m<ABD = _____
m<DBC = _____
(Algebraic answers)

12.



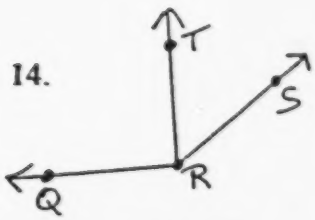
How many angles are in this diagram?___
Name them below.

13.



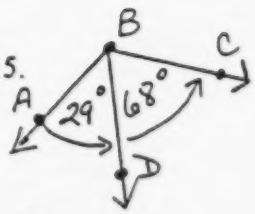
m<____ + m<____ = m<____

14.



m<QRT is 10 more than twice the m<TRS.
The m<QRS = 70.
x=_____, m<TRS=____
m<QRT =_____

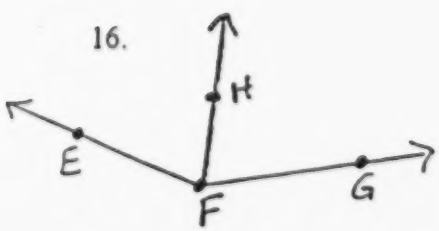
15.



$\overrightarrow{BD}$ bisects <ABC

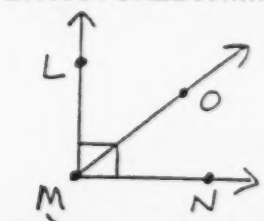
_____
(True or False)

16.



$\overrightarrow{FH}$ bisects <EFG.
m<EFH = 10x + 5
m<HFG = 11x - 3
x = _____, m<<EFH = _____
m<HFG = _____, m<EFG = _____

EXTRA CREDIT!!!!!!!!



$\overrightarrow{MO}$ bisects <LMN
m<LMO = _____
m<OMN = _____

Appendix G

Geometry Test
Chapter 1

Name_____

Part A: In column II, draw a diagram to represent the information given in column I. In column III, write an equation that can be derived from the information in column I.

COLUMN I	COLUMN II	COLUMN III
GIVEN INFORMATION	LABELED DIAGRAM	EQUATION
1. ℓ bisects $\overline{XY}$ at Z		
2. <a and <b are adjacent supplementary angles		
3. $\overrightarrow{TS}$ bisects <RTW		
4. $\overline{AB}$ is perpendicular to $\overline{DC}$. $\overline{AB}$ intersects $\overline{DC}$ at Q.		

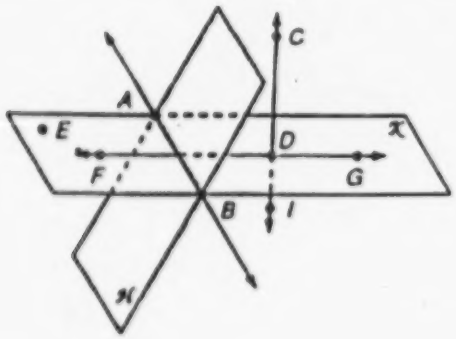
5. Circle the letter of the best answer.

A line that is perpendicular to a segment AND contains the segment's midpoint is defined as _____.

A. an obtuse line
B. an adjacent angle
C. a perpendicular bisector
D. a right bisector

Appendix G (Continued)

Use the diagram below for problems 6 through 9



6. Name two noncoplanar lines._____
7. Name three collinear points._____
8. $\mathcal{K}$ and $\mathcal{H}$ intersect to form a _____
9. Name $\overrightarrow{FD}$ in another way._____

Given points S(3,-5) and T(1,1)

10. Find ST

11. Find the midpoint of ST

Part B.

Evaluate the following problems.

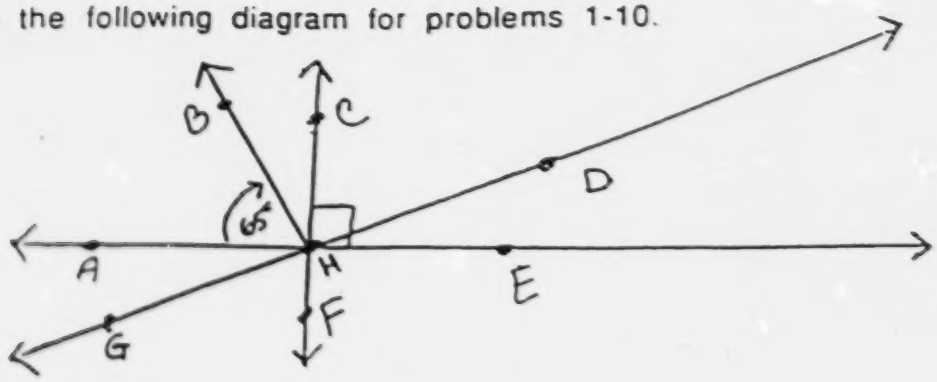
1. $\angle A = 46$ and $\angle B$ is the complement of $\angle A$
   Find the measure of $\angle B$._____
   Find the measure of the supplement of $\angle B$ _____

2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1$ is five times $\angle 2$
   $m\angle 1 =$_____ $m\angle 2 =$_____

Appendix G (Continued)

Part C.

Use the following diagram for problems 1-10.



1. Name 2 complementary angles._____

2. Name 2 supplementary angles._____

3 Name a point on the interior of <DHB._____

4 Name a point on the exterior of <GHE._____

5. Name a pair of vertical angles_____

6 m<BHC =_____

7 If $\overrightarrow{HD}$ bisect <CHE, m<BHD=_____

8. <GHC is a _____ angle (right, obtuse, acute)

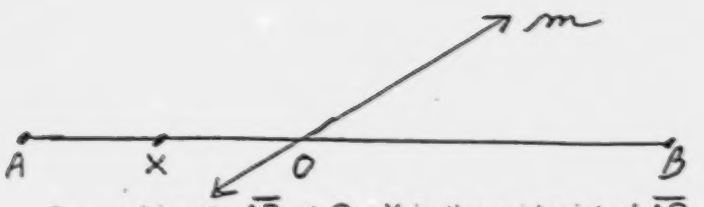
9. <GHE is a _____ angle (right, obtuse, acute)

10. <GHF is a _____ angle (right, obtuse, acute)

11. Name a pair of supplementary angles that are not a linear pair.

_____

Appendix G (Continued)



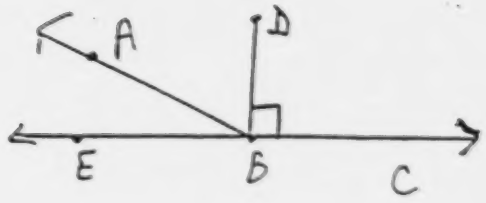
3. $\overleftrightarrow{m}$ bisects $\overline{AB}$ at O. X is the midpoint of $\overline{AO}$. OB=26.

AX=_____

XB=_____

4. m<ABD = x - 4, m<ABE = 10x + 6

m∠ABC=_____



EXTRA CREDIT

A. If the sum of the complement and the supplement of an angle equals 160 , what is the measure of the angle

B. $\overrightarrow{OA}$ bisects <BOC The m<BOA = 3x+3, and m<BOC = 9x-18. Find the measure of the supplement of <AOC

Appendix H

Geometry Quiz
Chapter 2

I. Underline the hypothesis and circle the conclusion of the following statement .

If I am not tired, then I can't sleep.

The statement above is called a _____ statement.

II. Write the following statements in if p, then q form.

1. Points that lie on the same line are collinear.

2. A right angle has a measure of 90 degrees.

3. Every linear pair of angles is supplementary.

III. Draw a counter example to show the following statements are false.

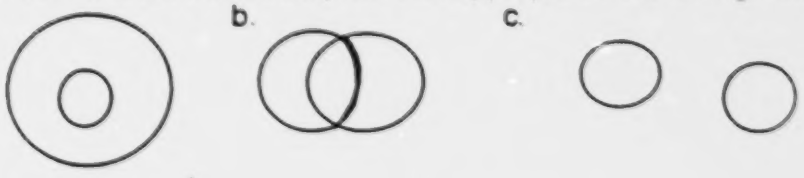
1. If AB=BC, then B is the midpoint of $\overline{AC}$.

2. If a number is divisible by 4, then it is divisible by 6.

3. If two angles are supplementary, then they are adjacent.

IV. Match each relationship with its appropriate Venn diagram.

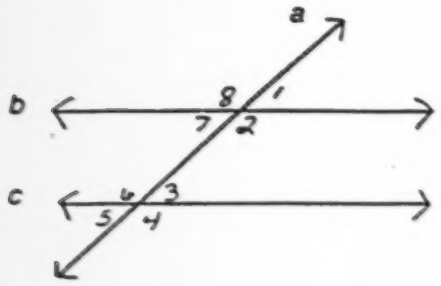
a. b. c.



1. Vertical angles and equal angles. ____
2. Right angles and acute angles. ____
3. Linear pair and supplementary angles. ____
4. Complementary angles and adjacent angles. ____
5. Supplementary angles and equal angles. ____

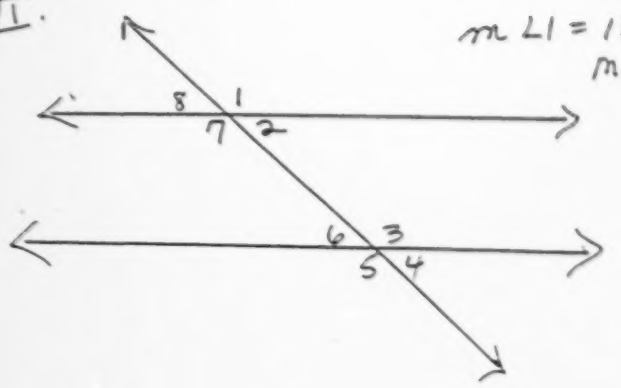
Appendix H (Continued)

V. Refer to the diagram below for the following problems.



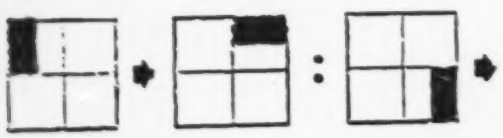
1. Which lines are parallel? _____
2. Which line(s) are transversal lines? _____
3. List a pair of corresponding angles. _____
4. List a pair of alternate interior angles. _____
5. List a pair of alternate exterior angles. _____
6. List a pair of vertical angles. _____

VI.



$m\angle 1 = 130^\circ$. FIND THE MEASURE OF ALL OTHER ANGLES IN THIS DIAGRAM.

VII. FIND THE NEXT TERM (OR PATTERN) IN THE SERIES.

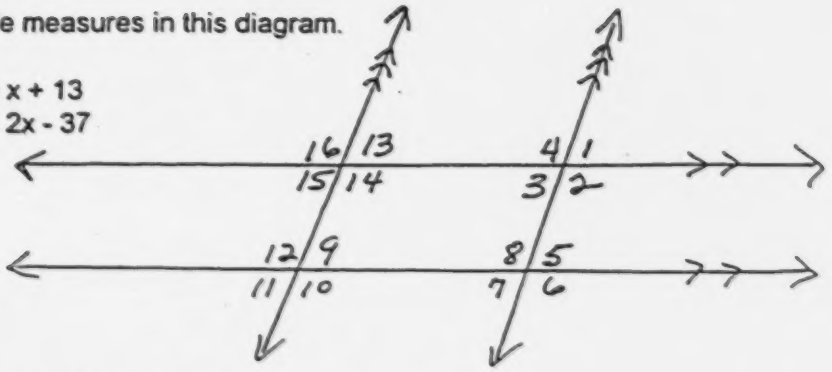
1. 1, 5, 14, 30, 55, 91, ___
2. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, ___
3. 

Appendix I

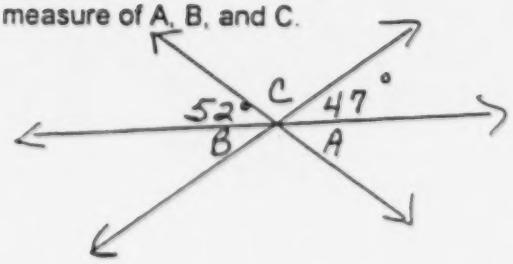
GEOMETRY
Chapter 2 Quiz 2 Name_____

1. Find all angle measures in this diagram.

m<1 = x + 13
m<3 = 2x - 37



2. Find the measure of A, B, and C.



m<A = _____
m<B = _____
m<C = _____

3. Write a program for the area of a rectangle. (HINT: A = l • w)

Appendix I (Continued)

4. Conditional: If two angles form a linear pair, then they are supplementary. _____(T/F)

Converse: _____(T/F)

Inverse: _____(T/F)

Contrapositive: _____(T/F)

5. Conditional: $p \Rightarrow q$ T

Converse: _____ _____(T/F)

Inverse: _____ F (T/F)

Contrapositive: _____ _____(T/F)

Appendix J

Name_____

Geometry
Chapter 2 Test

I. Write (A) always, (S) sometimes, or (N) never to make each of the following statements true.

1._____ Definitions can ____ be written in if and only if form.

2._____ A conditional is ___ a true statement.

3._____ The converse of a true conditional is ___ true.

4._____ The contrapositive of a true conditional is ___ true.

5._____ If two lines do not intersect, they are ___ parallel.

6._____ The complement of an acute angle is ___ obtuse.

7._____ If two angles form a linear pair, they are ___ supplementary.

8._____ If two angles are supplementary, they ___ form a linear pair.

9._____ Parallel lines ___ intersect.

II. Complete the following by filling in the blank.

1._____A line that intersects two or more coplanar lines in two distinct points is called a ___.

2._____ A statement in if-then form is called a ___.

3._____ A ___ is a statement that seems to be true.

Appendix J (Continued)

III. Determine whether the following conditionals are true or false. If false, draw a conterexample.

True/False Counterexample

1. _____ If <1 and <2 are complementary, then <1 and <2 are both acute.

2. _____ If <5 and <6 are equal, then <5 and <6 are vertical angles.

3._____ If PQ⊥PR, then <QPR is a right angle.

IV. Miscellaneous

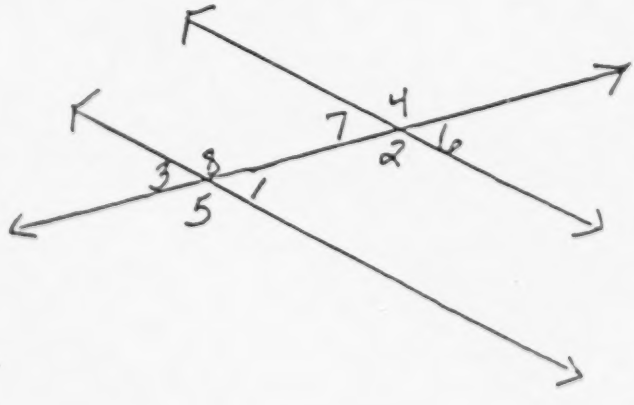
1. Underline the hypothesis and circle the conclusion of the following conditional:

If A mares eats oats, then little lambs eat ivy.

2. Write the following definition in if and only if form.
Perpendicular lines form right angles.

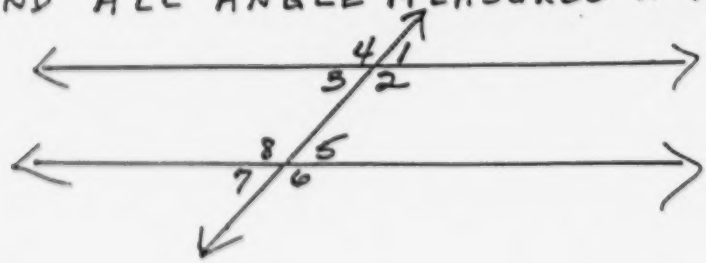
3. Use the figure at the right to identify the following angles (corr., vert., alt. int., etc...).

a. <1 and <5 _____
b. <4 and <5 _____
c. <2 and <8 _____
d. <2 and <4 _____
e. <1 and <6 _____



Appendix J (Continued)

V. FIND ALL ANGLE MEASURES IN THIS DIAGRAM



$m\angle 2 = x+25$

$m\angle 7 = x+17$

VI.

Given the following statements as true:

a. If you eat mud, then your face is dirty.
b. If you wear an earring, then your face is dirty.
c. If you are Mr. Clean, then you wear an earring.
d. If your face is dirty, then you will scare Mr. Clean.

Can you conclude that the following are true. Use (+) for true and (-) for false

A. _____ If you eat mud, then you will wear an earring.

B. _____ If you don't wear an earring, then you are not Mr. Clean.

C. _____ You scare Mr. Clean only if you eat mud.

D. _____ If you are Mr. Clean, then you will scare Mr. Clean.

VII.

1. If I can't understand a problem, I get confused while studying it.
2. If a problem doesn't follow a pattern then I can't understand it.
3. If I don't get confused while studying a problem, I can understand it.
4. If I get confused while studying a problem, then it is giving me trouble.
5. If a problem is giving me trouble, then it is too hard.

a. Use the transitive property for conditionals and arrange all the conditionals that apply in a logical order. List the correct order below. (use the numbers)

_____

b. State the resulting conditional. If _____

_____

Appendix J (Continued)

For each of the following, state the conclusion that can be drawn from the conditional and the information that follows. If no conclusion can be drawn, write "none".

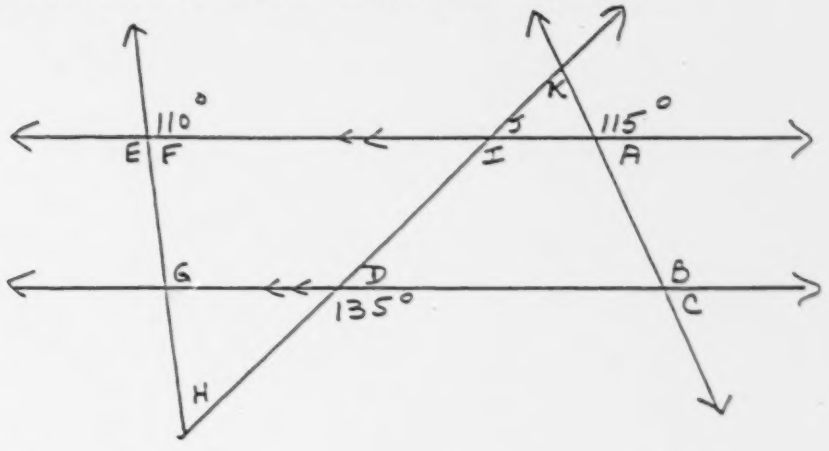
1. If two angles are vertical angles, then they have the same measure. <A and <B have the same measure.

conclusion: _____

2. If two angles form a linear pair, then the angles are supplementary. <1 and <2 form a linear pair.

conclusion: _____

IX. Find the measure of lettered angles A through K in the diagram.



m<A = _____ m<D = _____ m<G = _____ m<J = _____
m<B = _____ m<E = _____ m< H = _____ m<K = _____
m<C = _____ m<F = _____ m<I = _____

CHALLENGE PROBLEM

Conditional: _____ _____(T/F)

Converse: _____ T (T/F)

Inverse: not z ⇒ w _____(T/F)

Contrapositive: _____ F